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ABSTRACT

This paper offers a detailed review of the literature concerning sex differences in the learning of mathematics. It identifies cognitive, affective and educational variables which have been either shown or hypothesized to contribute to sex-related differences in mathematics learning. The author analyzes each study in detail. One important finding is that many studies crediting boys with more mathematical ability than girls were carried out on populations in which the boys had actually studied more math than the girls. The author believes that if the amount of time spent learning mathematics is equated for males and females, educationally significant sex-related differences in math performance will disappear. Much evidence is presented to support this belief. She concludes with recommendations in three areas: (1) directions for research; (2) intervention or demonstration projects; and (3) general considerations to optimize the chances that the money spent will accomplish its goals, i.e, to improve females' participation in mathematics now and in the future. (Author/BP)

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Influences of Selected Cognitive, Affective and Educational Variables on Sex-related Differences in Mathematics Learning and Studying

BEST COPY AVAILABLE

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October 1976

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
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Sex-related differences in mathematics learning is a topic that has received a great deal of attention in the lay press as well as in professional, educational and psychological literature for a number of years. However, much of what is considered as "truth" is based on myth, inadequate reporting, or research studies which failed to control one important variable, i.e., the number of years of mathematics studied while enrolled in high school. One must understand as clearly as possible if, when and where, sex-related differences exist in performance in the broad intellectual area called mathematics, before one can begin to hypothesize concerning related variables. Therefore, the first major goal of this paper is to identify what is believed about sex-related differences in overall mathematics learning. In relation to this, questions will be raised about the validity of these beliefs, and one specific area will be identified as the most salient cause of sex-related differences in mathematics.

The second major goal of this paper is to identify cognitive, affective and educational variables which have been shown, or have been hypothesized, to contribute to sex-related differences in mathematics

learning and studying. Two major bodies of professional literature will serve as the sources, i.e., the well documented mathematics education literature and the rapidly growing psychological literature related to the intellectual development of women. These two sets of knowledge all too often have been disjoint and it is necessary to synthesize relevant information from both sources. The importance of doing this is evident. Because of the nature of the subject matter, mathematics learning does not take place, other than incidentally, outside of formal schooling. To ignore what is known about why and how people learn mathematics as reported in math education literature would indeed be foolish. However, much of what is known about how people learn mathematics must be reevaluated in light of the new insights being generated by psychologists specializing in the study of women. These psychologists are reinterpreting old data and gathering new data as questions of integrity are being asked about women and their intellectual development. In some cases the synthesis of these two bodies of literature will be relatively easy, and clear-cut conclusions, testable hypotheses or plans for action will emerge. In other cases, conflicting evidence will be apparent and new data will need to be gathered before any plans for action can emerge.

SEX-RELATED DIFFERENCES IN THE LEARNING OF MATHEMATICS

For the purposes of this paper, learning and achievement in mathematics are used interchangeably. Both are indicated by a performance which results from the interaction of inherent and environmental variables. Such performance is usually indicated by scores on tests involving a

variety of mathematics at cognitive levels ranging from low level computational skills to high level problem solving skills. Care must be exercised in inferring from such scores to anything about inherent ability or environmental influences. Performance on a mathematics test is merely a response to a particular set of items at a specific point in time. Such responses are usually grouped together to describe a particular set of people by measures of central tendency and deviation. At a different point in time individuals within that group might respond somewhat differently but it is assumed that the group response would be somewhat similar. Nothing can be inferred, however, about why that group performed in that particular way. Keeping in mind this limitation, information about mathematical performance is useful. It is an indicator of how much learning has taken place and also is an efficient predictor of future success in mathematics.

Mathematics educators have used sex as a variable in research concerned with mathematics achievement for a number of years. Many general summaries have been published and most include something about comparative learning of math by males and females. There is a remarkable degree of consensus about sex-related differences in those reviews published before 1974. E. g., "The evidence would suggest to the teacher that boys will achieve higher than girls on tests dealing with mathematical reasoning" (Glennon and Callahan, 1968, p. 50). "From junior high school and beyond. . . boys now surpass girls in studies involving science and mathematics" (Suydam & Riedesel, 1969, p. 129). "Sex differences in mathematical abilities are, of course, present at the kindergarten level and undoubtedly earlier" (Aiken, 1971, p. 203).

Basically all these reviews concluded that while there might not be a sex-related difference in young children, male superiority was evident by the time learners reached upper elementary or junior high school. In addition, males were definitely superior in higher level cognitive tasks which assume increasing importance as one progressed to advanced mathematical study, i.e., Algebra, Geometry and beyond. In none of these reviews was the magnitude or the educational significance of the differences discussed and no concern for explanatory reasons or remedial measures were evident (Fennema, 1974).

The reviews of sex-related differences in math published since 1974, do not always draw the conclusion of male superiority in mathematics. Fennema (1974) reviewed 36 studies and concluded that there were no sex-related differences in elementary school children's math achievement. She also found little evidence that such differences exist in high school learners. However, there was a trend that males excelled in higher level cognitive tasks and females in lower level cognitive tasks. Fennema further concluded that more research is needed on this issue since previous research has raised more questions than it had answered. Callahan & Glennon (1975) agree with this conclusion. Maccoby and Jacklin (1974) in a highly quoted review disagree. They state that one "sex difference that /is/ fairly well established . . . is that boys excel in mathematical ability" (p. 351-352).

In order to clarify the reality of sex-related differences in math, four major studies of sex-related differences in math achievement must be specifically noted: Project Talent, the National Longitudinal Study of Mathematical Abilities (NLSMA), the National Assessment of Educational

Progress (NAEP), and the Fennema-Sherman Study. The first three of these studies have had much impact on current belief about sex-related differences and as such should be examined carefully. The fourth study, results from which are in press, is the most current information available on sex-related differences.

Data for Project Talent were gathered about 1960 (Flanagan, et al., 1964). This study assessed math achievement (among many other things) of a random sampling of high school students in the United States ($n \approx$ 440,000). Mean scores for both sexes were very low. In Grade 9 sex-related differences were negligible but by 12th grade males tended to do better. The mean difference at the 12th grade, while statistically significant, appears to have little educational significance (approximately one item). No attempt was made to control the number of mathematics courses subjects had taken previously. Higher percentages of males than females were enrolled in college preparatory courses so males undoubtedly had taken more math courses. Therefore, a population of males with more mathematical background was being compared with a population of females with less mathematical background.

In 1975 a small follow up study to Project Talent was done by testing approximately 1800 students in Grades 9-11 in seventeen of the original schools (1976). After careful statistical checks on reliability of the comparisons and adjustment for any change in school SES the following conclusions were made: 1) While the math test scores were fairly stable from 1960 to 1975, the difference between male and females scores had been reduced with males scoring .4 of an item higher in 1975. 2) On computation tasks male scores had declined 17% and

female scores 11% with the female mean score being 8.2 points higher than the male mean score. 3) Quantitative reasoning scores declined 8% for each sex with females scoring .6 of an item lower in 1975. It is difficult after carefully examining this data from Project Talent to conclude that males math achievement was higher than that of females in 1960 or 1975.

Support for the belief that females do not achieve as well as males in math could come from the NLSMA data which were gathered during 1962-67. In these multitudinous studies, sex was used as a control variable. Analyses were done independently by sex whenever significant sex by any other variable interaction was found. Unfortunately, the results from these studies have been inadequately reported and interpreted making the knowledge they could contribute to the area under consideration largely unavailable. However, a summary statement says: "Differences favoring girls were for variables at the comprehension level (the lowest cognitive level tested) and the differences favoring the boys were for variables at the application and analysis level"(Wilson, 1972, p. 95). The directors of this federally financed program abrogated their responsibility to females when they followed the above remarks with this statement: "Interpretation and comment on this pattern will be left to persons involved in the women's liberation movement"(Wilson, 1972, p. 95). The number of math courses which had been taken previously by the subjects in the NLSMA studies was controlled so the conclusion reached undoubtedly was statistically valid in 1967. What is unknown is the size of the difference between the mean male and female performance scores and the educational significance of that difference.

Results from the 1972-73 mathematics data collection of NAEP have received much publicity and one sentence has been widely quoted: "In the mathematics assessment, the advantage displayed by males, particularly at the older ages, can only be described as overwhelming" (Mullis, 1975, p. 7). Inspection of this data confirms that males did outperform females at ages 17 and 26-35. However, at ages 9 and 13 differences were minimal and sometimes in favor of females. The problem of comparable populations is a concern here also. The population was selected by sophisticated random sampling techniques with no control for educational or mathematical background. Since males have traditionally studied mathematics more years than have females, once again a population of males with more background in math was being compared with a population of females with less background in math. At ages 9 and 13 when the educational and mathematical background was similar, the achievement of both sexes was also similar.

The Fennema-Sherman Study, data for which were collected in 1975-76, investigated math achievement in grades 6-12. (Fennema and Sherman Note 1; Sherman and Fennema, Note 2; Fennema and Sherman, Note 3.) This National Science Foundation sponsored study investigated a variety of levels of mathematical learning as well as cognitive and affective variables hypothesized to be related to differential mathematics achievement by males and females. The results of this study have wide generalizability because of the diverse carefully selected population. In grade 9-12 (n = 1233) with subjects whose math backgrounds were carefully controlled, significant differences in achievement in favor of males (approximately two items) were found in two of four schools. In grades

6-8 (n = 1330) significant differences were found in favor of females in a low level mathematical cognitive task in one of four tested school areas. In another of the four school areas significant differences were found in favor of males in a high level mathematical cognitive task.

The discussion to this point has considered mathematical achievement as if it were a precisely defined skill or attribute. This is not so. The measures of achievement utilized, with the exception of those used in NLSMA, have been a mixture of several types of mathematical performance. There is, however, one body of literature that deals with a rather specific type of mathematics performance which is important in the understanding of sex-related differences in mathematics. That literature focuses on the process of mathematical problem solving.

The problem-solving process is important at any mathematical level and assumes increasing importance as one moves into advanced mathematics. In contrast to performing rote, computational exercises one must be able to abstract relevant data, translate the problem into mathematical terms, decide what mathematical process(es) will solve the problem and perform these processes accurately. Schonberger (1976) thoroughly reviewed many problem solving studies which used sex as a variable. She concluded that while sex-related differences in problem solving abilities may be small, male superiority does appear to exist starting at early adolescence and increasing until maturity. Schonberger's own study, did not support these conclusions as she found no sex-related differences in math problem solving in seventh grade subjects.

What, then, can be concluded about sex-related differences in math learning in 1976?

1. There are no sex-related differences evident in elementary school years. This is at all cognitive levels from computation to problem solving. This conclusion has been accepted for a number of years.
2. After elementary school years, differences do not always appear.
3. Starting in about grade 7, if differences appear, they tend to be in the males' favor, particularly on tasks involving higher level cognitive skills.
4. There is some evidence that sex-related differences in math learning in high school may not be as large in 1976 as they were in previous years.
5. Conclusions reached about male superiority have often been gathered from old studies or from studies in which the number of math courses taken was not controlled. Therefore, a better mathematically educated group of males was being compared to a group of females who had participated in less mathematics education. In reality, what was being compared were not males and females but students who had studied math 1-3 years in high school with students who had studied math 2-4 years in high school.

SEX-RELATED DIFFERENCES IN THE STUDYING OF MATHEMATICS

There are sex-related differences in the studying of mathematics. This is indicated by females choosing not to enroll in mathematics courses in high school and by the paucity of females in university math courses. Undoubtedly, the most serious problem facing those concerned with equity in mathematical education for the sexes is ensuring that females continue their study in mathematics. In support of this statement consider some data from Wisconsin. During the 1975-76 academic year, while approximately the same number of females and males were enrolled in Algebra, in the advanced courses many more males were enrolled (see Table 1).

Table 1

Number of Males and Females Enrolled in Wisconsin in Mathematics Courses[a]

Course	Males	Females
Algebra[b]	41,404	41,579
Geometry	20,937	20,280
Algebra II	11,581	9,947
Pre-Calculus	3,234	1,917
Trigonometry	4,004	2,737
Analytic Geometry	1,752	970
Probability/Statistics	1,113	581
Computer Mathematics	3,396	1,481
Calculus	611	262

[a]Data obtained from Wisconsin Department of Public Instruction Enrollment Statistics, 1975-76.

[b]Students enrolled in one year course and two-year course.

Although only symptomaticof the effects of many variables, electing not to study math in high school beyond minimal or college requirements is the cause of many females' nonparticipation in math related occupations. The one variable which can be positively identified as causing sex-related differences in math is the differential number of years females and males spend formally studying and using mathematics. Such a simplistic explanation of such an important problem seems too good to be true. However, this author believes strongly that if the amount of time spent learning mathematics is equated for males and females, educationally significant sex-related differences in math performance will disappear.

A number of researchers are addressing the issue of equality of educational opportunity in relation to quantity of time spent in school. Harnischfeger and Wiley (1975) present the clearest explication of a model which shows the relation between pupil activities in school and academic achievement. Harnischfeger and Wiley believe that there is a strong relationship between academic achievement and hours of instruction. They state:

> In a school-level regression analysis, we estimated the coefficients linking verbal, reading, and mathematics achievement to number of hours of instruction per year with adjustment for pupil's prior characteristics. In terms of typical gains in achievement over a year's period we concluded that in schools where students receive 24 percent more schooling,

> they will increase their average gain in reading comprehension by two-thirds and their gains in mathematics and verbal skills by more than one-third. These tremendous effects indicate that the amount of schooling a child receives is a highly relevant factor for his achievement (Wiley and Harnischfeger, 1975).

While the effect of non-election of math courses is the most important cause of sex-related differences in the learning of math, explicit information concerning this is unavailable. Answers to the following questions are essential.

1) What is the scope of the problem nationwide? Are there geographical, socioeconomic, or race differences in the election of mathematics courses?
2) Are there identifiable geographical areas or schools where differences exist in the election of math courses by females? What are the characteristics of areas and schools where either an unusually high or low percentage of females elect to study math beyond minimal requirements? Casserly (1975) offers some information regarding this question.
3) Are there sex-related differences in the type of math courses being taken by the sexes? Are females taking more terminal or applied courses such as "consumer" math while males take courses that lead to more advanced study?

How to change females' election of mathematics courses is unknown. In fact, changing any behavior in a systematic way appears to be an elusive, if not impossible occurence. It has been simplistically assumed that the first step in changing human behavior is to identify the causative factors of that behavior. Since the relationship between a factor and a behavior can not usually be directly observed, and in spite of the cautionary remarks by statisticians, causative factors are usually identified as those which occur in juxtaposition with the behavior under consideration. After these so called causative factors have been identified, many believe that one has only to modify them and the behavior automatically changes in the desired way. Although this chain of inferences has some validity, there are some problems with it. Not all factors related to performance can be modified. For example, there are many studies which indicate that mothers' educational attainment is related to daughters' educational aspirations. It is obvious that while this piece of information is interesting, mothers' educational attainments are almost impossible to change.

Human behavior can not be partitioned into specific factors for other than theoretical purposes. Even when modification in an identified factor is possible, the factors exist in such a complex network of intersecting factors that modification of one is ineffective in producing long range behavioral change. Some related factors, while perhaps amenable to change, either can not be changed by schools or the schools' possible impact is limited. For example, some data indicate that fathers have a direct influence on girls' performance in the cognitive areas.

The possibility of schools changing fathers' treatment of girls is either impossible or so prohibitively expensive in time and resources to render it impossible.

It is only sensible to concentrate the efforts of NIE in areas in which change may be effected. Therefore, this paper will concentrate on variables which are related to females' performance in math and which are most amenable to change by intervention techniques within the framework of formal education. Since the concern here is with ensuring equity in mathematical education of females and males, emphasis will be upon identifying those areas related to math performance in which sex-related differences have been found.

COGNITIVE VARIABLES

"Mathematics is essentially cognitive in nature; and the principle, distinguishing goals or objectives of mathematics instruction are (and should be) cognitive ones" (Weaver, 1971, p. 263). Since mathematics is a cognitive endeavor, the logical place to begin to look for explanatory variables of sex-related differences in math performance is in the cognitive area. It is within this area that the most important variable can be found, i.e., the amount of time spent studying mathematics. This variable has already been discussed.

In addition to total amount of time spent in studying mathematics, what one studies is also important. One learns what is practiced. If one practices computational algorithms (a low level cognitive task) one learns to compute accurately and quickly. If one practices

solving problems (a high level cognitive task) one learns to solve problems. Since there appears to be sex-related differences in levels of cognitive skills in mathematics, can one assume that females tend to spend more time doing one level of cognitive task while males tend to do another level of task? This would be easy to ascertain at the high school level by finding out if females and males tend to take different types of math courses. It is also of vital importance to ascertain if elementary school males and females spend their time in math classes doing somewhat different activities.

Spatial Visualization

One cognitive variable that may help explain sex-related differences in mathematical achievement is spatial visualization--a particular subset of spatial skills. Spatial visualization involves visual imagery of objects, movement or change in the objects themselves or change in their properties. In other words objects or their properties must be manipulated in one's "mind's eye" -- or mentally. Even though the existence of many sex-related differences is being challenged, the evidence is still persuasive that in the American culture male superiority on tasks that require spatial visualization is evident beginning during adolescence (Fennema, 1975; Maccoby & Jacklin, 1974).

The relationship between mathematics and spatial visualization is logically evident. In mathematical terms spatial visualization requires that objects be (mentally) rotated, reflected and/or translated. These are important ideas in geometry. In fact James and James (1968) in defining geometry as "the science that treats of the

shape and size of things . . .the study of invariant properties of given elements under specified groups of transformations" (p. 162) are describing accurately most conditions which are met by items on spatial visualization tests.

Many mathematicians believe that all of mathematical thought involves geometrical ideas because the total discipline of mathematics can be defined as the language for describing those aspects of the world which can be stated in terms of "configurations" (Bronowski, 1947). Meserve (1973) believes that each person who makes extensive use of all areas of mathematics uses the modes of thought of geometry at every turn and that "even the most abstract geometrical thinking must retain some link, however attenuated, with spatial intuition" (p. 249). In the Russian literature, mathematics and spatial abilities are regarded as inseparable (Kabanova-Meller, 1970). Therefore, if spatial visualization items are geometrical in character and if mathematical thought involves geometrical ideas, spatial visualization and mathematics are inseparately intertwined.

Not only are spatial visualization components an integral part of the structure of mathematics, but spatial representations are being increasingly included in the teaching of mathematics. E. g., the Piagetian conservation tasks, which are becoming a part of many pre-school programs, involve focusing on correct spatial attributes before quantity, length, and volume are conserved. Most concrete and pictorial representations of arithmetical, geometrical and algebraic ideas appear to be heavily reliant on spatial attributes. The number

line, which is used extensively to represent whole numbers and operations on them, is a spatial representation. Commutativity of multiplication illustrated by turning an array 90 degrees, involves a direct spatial visualization skill. Many other examples could be cited.

Although the relationship of mathematics and spatial visualization ability appears logical, empirical data confirming a positive relationship is less clear. Many factor analytic studies have explored this relationship and several authors have reviewed the literature. Some investigators have definitely concluded that spatial skills and learning of mathematics are not related. In 1967, Very concluded: "Research on spatial ability has failed to produce any significant correlation of (the spatial factor) with any facet of mathematics performance" (p. 172). Fruchter (1954) stated that "spatial ability is unrelated to academic performance with the possible exception of a few very specialized courses such as engineering drawing" (p. 2). Smith (1964) concluded that although "there are several studies which indicate consistently that spatial ability is important in tests which are genuinely mathematical as distinct from those which involve purely mechanical or computational processes . . . the question whether the mathematical ability is dependent on the visual factor (or factors) has not been definitely answered" (p. 127, 68).

Even in the specialized mathematical area of geometry where one would expect to find the strongest relationship, empirical findings do not indicate clearly that the two are related. Lim concluded in 1962 after a thorough review of relevant literature that the

evidence for a relationship between geometric ability and spatial visualization was inconsistent and unreliable. Werdelin (1961) was not willing to conclude definitely that empirical data indicate that spatial visualization ability and geometry ability were related. However, he felt that "there is strong pedagogical reason to believe in a connection between the ability to visualize and geometric ability" (p. 39).

Other authors feel that data indicate a positive relationship. In 1951, Guilford, Green and Christensen concluded that spatial visualization ability helped in solving mathematics problems. French (1951, 1955) showed that successful achievement in mathematics depends to some extent on use of spatial visualization skills. In a recent review, Aiken (1973) concluded that spatial-perceptual ability was one of the "most salient" mathematical factors extracted in various investigations. Obviously, the relationship between learning in mathematics and spatial ability is not clear and the need for more data is great.

Even less is known about the effect that differential spatial visualization has on the mathematics learning of females and males. Indication that the relationship between the learning of mathematics and spatial visualization is an important consideration, is the concurrent development of sex-related differences in favor of males in mathematics achievement and spatial visualization skills. No significant sex differences in either mathematics achievement nor spatial visualization skills have been consistently reported in subjects 4-8 years old. Sex differences in performance on spatial visualization tasks

become more pronounced between upper elementary years and the last year of high school and the differences show a pronounced increase during this time span (Maccoby & Jacklin, 1974). Sex differences in mathematical achievement that do exist also appear during this time span (Fennema, 1974).

It appears reasonable, therefore, to hypothesize that since there is a concurrent developmental trend and since tests of spatial visualization contain many of the same elements contained in mathematics, the two might be related. Perhaps less adequate spatial visualization skills may partially explain sex-related differences in achievement in mathematics.

The Fennema-Sherman study specifically investigated the relationship between mathematics achievement and spatial visualization skills. These data do not support the idea that spatial visualization is helpful in explaining sex-related differences in math achievement In this study of males and females (grades 6-12) enrolled in math courses, few sex-related differences in either math achievement or spatial visualization skills were found. The two were related ($r = .5$) similarly for both sexes and spatial visualization appeared to influence both males and females equally to continue studying math.

Several areas should be investigated to clarify the relationship between spatial visualization skills and females' performance in math:

1) What is the impact of spatial visualization skills on females learning of geometry and their election of additional math courses?

The big dropoff in the number of females in math courses occurs after 10th grade, when students are enrolled in traditional Euclidean geometry classes. Although no precise data is available, students appear to view geometry classes differently than they view other math classes. This view of geometry may be a strong influence on deciding whether more math courses are taken. More information is needed related to this: What influence does geometry have on the decision to elect more math courses? Because of the spatial visualization component of geometry, are weaker spatial visualization skills a determining factor of success in this area? There has been some research conducted in this area but none regarded as definitive. In relation to geometry, another area needs investigation. Is the deductive component of geometry a factor in females' success or election of more math courses?

2) What is the effect of spatial skills on mathematical learning at various developmental levels?

Smith in 1964 hypothesized that while spatial ability may not be related to mathematics ability at beginning stages of mathematics learning, advanced mathematics learning increasingly depends upon spatial ability (Smith, 1964). It would appear that this hypothesis was made after surveying a number of studies which used high school or college students as subjects and relatively sophisticated mathematical ideas as criterion measures. Little or no data were presented from studies with younger learners. However, in 1964 one could have built a strong argument to support the idea that spatial ability was not related to mathematics at beginning stages of mathematics learning. Little or no

geometry was taught at the pre-high school level and most pre-high school mathematics tests focused primarily on arithmetical/computational ideas. Werdelin (1958) found computation to be negatively correlated with spatial ability. Therefore, Smith's hypothesis that spatial ability was not related to mathematical ability at beginning stages of mathematics learning was believable in 1964. The tests used to measure mathematics achievement probably included few items relevant to spatial ability and as such reflected the mathematics program of most schools previous to 1964.

Since 1964 a major change has taken place in most K-12 mathematics curricula. Geometry has become an integral part of the entire mathematics curriculum. More emphasis is being placed on the structure of math and increasingly mathematics is taught as an interrelated system of ideas. In order to learn new ideas, learners are dependent upon the presence of prerequisite ideas in their cognitive structure. Little is known about the importance of spatial ability in the acquiring of these prerequisite mathematical ideas upon which all advanced mathematical knowledge is based. The influence of spatial ability on learning elementary mathematical ideas may be of the utmost importance.

Developmental psychologists patterned on Piaget have theorized that at different stages of cognitive development certain modes of thought predominate and ideas are added to one's cognitive structure by utilizing actions, symbols which represent those actions, and symbols alone, in somewhat different blends. According to this theory, mental structures are formed by a continual process of accommodation to and assimilation of the environment. This adaptation (accommodation and

assimilation) is possible because of the actions performed by the individual upon the environment. These actions change in character and progress from overt, sensory actions done almost completely outside the individual; to partially internalized actions which can be done with symbols representing previous actions; to complete abstract thought done entirely with symbols. Thus, development in cognitive growth progresses from the use of physical actions to form schemas to the use of symbols to form schemas, i.e., learners change from a predominant reliance upon physical actions to a predominat reliance upon symbols.

Mathematical educators have increasingly accepted this theory of cognitive development and have translated it into educational practice by an increased emphasis upon the instructional use of three modes to represent mathematical ideas, i.e., concrete (enactive), pictorial (ikonic), and symbolic. Such a belief suggests that the blend of the usage of these representational modes should reflect the cognitive developmental level of the learner. Particularly at early stages of mathematical learning it is important to provide learners with concrete representations of mathematical ideas while symbols assume increasing importance as learners mature and mathematical ideas become more complex.

Most concrete and pictorial representations of mathematical ideas include spatial attributes, some of which are relevant to the mathematical idea being taught and some of which are not. The only way to add mathematical ideas to one's cognitive structure at early developmental levels is by interaction with concrete or pictorial materials

which represent those ideas, and since those representations depend heavily on spatial attributes, if one is hampered in perception of those spatial attributes then one is hampered in learning those early mathematical ideas. Without knowledge of these ideas, it is impossible to learn advanced mathematics. Therefore, spatial visualization appears to be important at early stages of learning.

Sherman (1967) has suggested that boys outperform girls on spatial tasks because they develop their spatial skills by participating voluntarily in more spatially oriented activities. Girls learn to read more easily than do boys. Because of ease of use of symbols, i.e., reading, do girls voluntarily, or are they encouraged to rely more heavily on symbols to learn mathematics rather than using concrete or pictorial representations? If so, perhaps inadequate usage of spatial representations may hamper both the development of their spatial skills and more advanced mathematical learning. No data is available to give insight into this problem. Empirical data from studies dealing with the use of various representational modes are not conclusive even about the value of concrete and pictorial representations. As far as is known no study has included spatial visualization as a factor. Certainly, more data are needed.

3) What is the interaction effect of spatial visualization ability and other abilities on achievement in mathematics?

Werdelin (1961) showed that girls were able to prove verbal theorems better than boys but were less able to translate words into figural images and then to transform those images in a directed way. Another truism, which is still accepted by Maccoby and Jacklin (1974),

is that females' verbal ability is more highly developed than is males'. Does the development of verbal ability in some way interfere with development of spatial ability? Werdelin (1958) in a factor analytic study found one spatial-visual factor in high school students which was related to a factor he called mathematical reasoning. Interestingly, he found the correlation between his spatial-visual factor and a numerical (or computational) factor was negative. Females often score higher on tests of computation than do males. Perhaps higher development of numerical or computational ability interferes with development of spatial-visualization ability. Both of these questions appear related to the earlier one of the impact of spatial ability on early mathematical learning. Does facility with symbols--computational or verbal--interfere with development of spatial skills?

4) What is the effect of possessing a greater variety of well developed abilities on mathematics learning?

Harris and Harris (1972), Werdelin (1958), and Very (1967) have shown a larger number of space factors for males than for females. Werdelin (1961) concluded that if one could attack a problem either verbally or spatially, one would be more apt to be able to solve it. His data showed that boys were superior on items which measured the ability to comprehend the organization of a visual figure and to reorganize it. Where items could be solved by verbal means and did not require that the problems be translated into a mental figure, no sex differences were found. If males have developed more abilities than have females they are able to attack mathematical problems in a variety of ways and thus score higher on mathematical achievement tests.

5) What sex differences in mathematics achievement would be found if spatial visualization were not a factor?

Tittle (1973) has shown that many tests commonly used to measure achievement are sexually biased. If a mathematics test contains many items requiring spatial skills in their solution, females possibly will not do as well as will males. It would be very interesting to construct a test that had little or no spatial content in it and compare the sexes on achievement. Perhaps no differences will be found if the test content is controlled in the spatial area. On the other hand, spatial visualization may be such an integral part of higher mathematical thinking that eliminating spatial aspects of mathematics tests too narrowly restricts the area of mathematical thinking. This aspect should be investigated.

Cognitive Style

Although not traditionally viewed as a cognitive variable, cognitive style utilized in the learning of mathematical content as well as in development of problem solving strategies appears to be important in understanding sex-related differences in math. Cognitive styles, based on two somewhat different aspects need to be considered, i.e., global-analytical and spatial-verbal. One appears not to be a useful avenue of investigation while the other does.

Witkin has been the foremost expositor of a global-analytical dichotomy in perception and a thorough review of his work can be found in Kagan and Kogan (1970). Witkin has consistently reported sex-related differences in tests measuring global-analytical skills. Since

mathematics is noted for analytic thinking (e.g., Analytic Geometry), performance on tests measuring something called analytic ability has wide appeal as an explanation of less adequate female mathematical learning. However, this relationship appears to be a semantic relationship at best. Analytic in one case does not mean the same as analytic in the other. The analytic ability described by Witkin appears to involve performance on perception tasks and the analytic ability required in math involves a high level cognitive task with as yet poorly understood dimensions. Therefore, the analytic-global dichotomy suggested by Witkin does not appear to be a useful tool in understanding problem solving strategies or the learning of mathematics (Schonberger, 1976). In addition, one of the more important components of the analytic-global tasks affecting mathematics learning may be the spatial dimension included in such tasks (Sherman, 1967).

One cognitive style dimension which may help in understanding sex-related differences in mathematics is the blend of spatial-verbal skills utilized in mathematical problem solving. Many mathematical problems can be solved by the use of symbols and/or by the use of figures or drawings. For example, consider the problem shown in Figure 1.

Insert Figure 1 about here

This problem can be solved by counting the squares and half squares included in the figure. Thus $1 + 1 + 1/2 + 1/2 + 1/2 + 1/2 = 4$ square units. The problem can also be solved by mentally moving the

What is the area of the figure enclosed by the heavy lines.

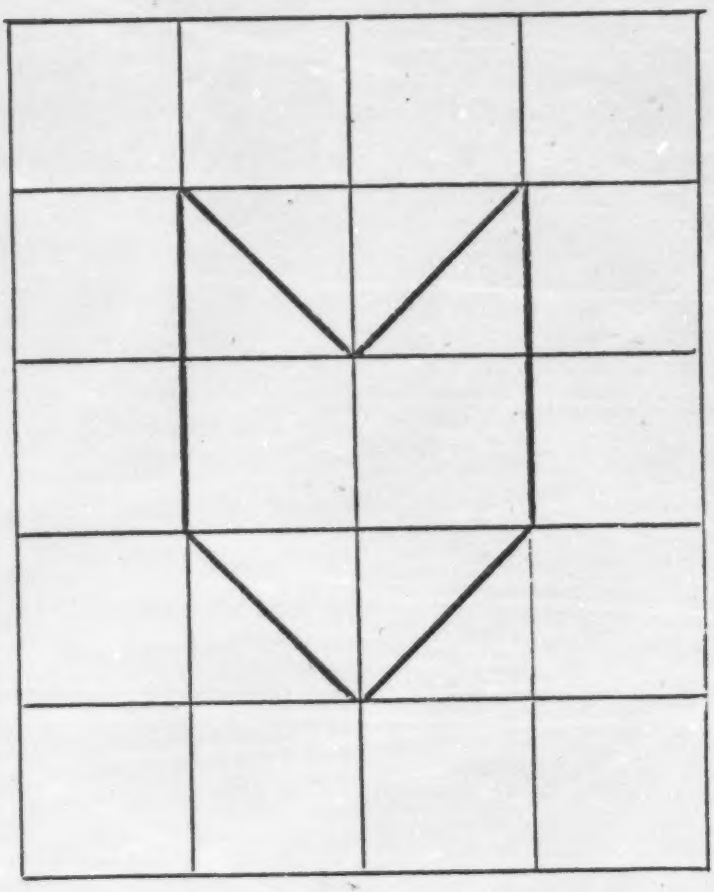

Figure 1.

half squares at the top or bottom of the figure so that a 2 x 2 square results which has an area of 4 square units. In both solutions, verbal skills and spatial skills are utilized. However, the blend is somewhat different with verbal skills used predominantly in the first solution and spatial skills used predominantly in the second solution.

It has already been said that little is known about how spatial visual skills are utilized in learning mathematical ideas. Even less is known about how or if, the verbal-spatial dimension of cognitive style is utilized in solving mathematical problems: It seems reasonable that there may be sex-related differences in the use of this cognitive style. Such differences may be an important contributor to differences in math studying by the sexes. This area needs investigation.

General Intelligence and Components of Mathematical Ability

Without a doubt the major intellective factor related to mathematics learning is general intelligence as defined by any number of intelligence tests. However, sex-related differences are not apparent from these tests (Maccoby & Jacklin, 1974). Such ability does not help explain sex-related differences in math performance (For a thorough review of this, see Armstrong, 1975).

Another approach to understanding math ability has been an attempt to identify its components through factor analytic techniques. Sex has been used as a variable in many such studies. Synthesizing these studies is difficult. Either no sex-related differences in factor structures have emerged or reported differences have not been

replicated. Aiken (1971) has done a review of factor analytic studies dealing with mathematical ability and this review reflects the knowledge base in this area. He concludes that "inter-individual and sex differences in mathematical abilities are, of course, present at the kindergarten level and undoubtedly earlier" (p. 204). However, he does not specify what these differences are or indeed cite studies which confirm that differences exist in any consistent pattern. Many of these studies appear to have been carefully designed, implemented and reported, but at the present time information gained from the studies has not helped in understanding sex-related differences in math. It is difficult to believe that further investigation in this area would be useful.

Verbal Variables

Verbal ability as measured by language tests of various kinds (vocabulary, spelling, grammar, reading, etc.) correlates highly with mathematical performance (Aiken, 1971). However, research relating this variable and mathematics does not provide insight into sex-related differences in mathematics performance because sex-related differences in verbal ability favor females. (Maccoby & Jacklin, 1974). Since verbal factors are so important to mathematics, why don't females achieve better in mathematics than do males? Little or no data are available to answer this intriguing question.

One important area could be considered. Perhaps the verbal factor related to mathematics is somewhat different from the verbal factor traditionally examined. In order to learn or to use mathematics

one must be facile with mathematical language. Language is vital to mathematics. Starting with numbers and numeration and continuing into the basic mathematical operations of addition, subtraction, multiplication and division, mathematics has a highly specific symbolism and syntax which must be learned. As one progresses to higher mathematical levels, symbolism is increasingly used to express ideas. This symbolism is unique to mathematics; it is isolated within mathematics; and without a thorough knowledge of it, abstract mathematical thought is impossible. It would appear that while the verbal factor in mathematics might be very similar to the verbal factor of words, there are also differences. Conciseness is one difference. One mathematical symbol is equivalent to many words resulting in a high concept density in mathematics writing. Another difference is the learning of the symbols. Learners are embedded in a social milieu which constantly bombards them with words. Not so with mathematical symbols. Their use is restricted to a great degree to mathematics classes in schools. Whether or not such a specialized verbal factor can be identified is unknown, but it appears reasonable that such a factor exists and that it might provide some understanding of sex-related differences in mathematics study.

AFFECTIVE VARIABLES

Most educators believe that affective variables are important contributors to the learning of mathematics and the relationship between the two has been the object of many studies. Attesting to this is the availability of several excellent reviews (Aiken, 1970a; Suydam and Weaver, 1975; Callahan and Glennon, 1975; Aiken, 1976). However, conclusions reached in these reviews appear to be contradictory. Suydam and Weaver (1975) summarizing elementary school studies state: "There is no consistent body of research evidence to support the popular belief that there is a significant positive relationship between pupil attitudes toward mathematics and pupil achievement in mathematics. . . . We have little research basis for believing that these two things are causally related" (p. 1-3). Also reviewing elementary school studies for the same age Callahan and Glennon (1975) agree with Suydam and Weaver. They conclude that the state of the art "makes it difficult to present compelling research evidence . . . that positive attitudes play an important role in contributing to mathematics achievement" (p. 80). Aiken (1976) states: "When attitude scores are used as predicators of achievement in mathematics, a low but significant positive correlation is usually found" (p. 295) at the elementary, secondary, college undergraduate and postgraduate levels.

Part of the contradictory conclusions can be explained by the age of the subjects being considered in the reviews. Two reviews (Suydam and Weaver, 1975; Callahan and Glennon, 1975) were basically

concerned with children in Grades 1-6. Problems of assessing attitude in these grades have not been addressed adequately and lack of carefully designed measuring instruments may have caused reviews to seriously question any significant differences reported. Aiken, in his review, was concerned with a much broader age spectrum. Even while recognizing the serious problems connected with the studies of young children, he was willing to accept the evidence of having some validity because the results coincided with studies having older subjects.

The most serious problem presented by the work done in this area is the definition used of the construct "attitude". One flaw is the impreciseness of the definition. For example, Callahan and Glennon define an attitude as children's liking or disliking of math. Aiken in one place (1970a) offers a definition basically in agreement with Callahan and Glennon and in another (Aiken, 1970b) defined attitudes as "variables which are not explicitly measured by tests of ability" (p. 28). (One wonders if the color of hair of the subjects might be an attitude.) What is the actual dimension that has been measured in studies dealing with attitudes? Does it involve euphoria, depression, confidence in performing the task, belief that mathematics is useful, recognition of success from peers, anxiety, or sense of failure? These dimensions are quite often included in a single scale which purports to measure "attitude".

Another serious flaw in many attitude studies is the global definition of mathematics used. Mathematics is a complex discipline involving many kinds of related but diverse subject matters and skills. To

assume that a person feels the same toward each part of mathematics is not reasonable. For example, computing the answers to 50 three digit by three digit multiplication problems could easily arouse different feelings in a person from those aroused when solving a mathematical puzzle.

Taking into consideration the problems not addressed by the research reported to date, several tentative conclusions can be reached.

1) There is a positive relationship between attitude and achievement which seems to increase as learners progress in school.
2) Attitudes toward mathematics are fairly stable--particularly about the sixth grade, although one longitudinal study showed a marked decrease from 6th grade to 12th grade (Anttonen, 1969).
3) Grades 6-8 seem to be critical in the development of attitudes.
4) Extremely positive or negative attitudes appear to be better predictors of achievement than more neutral feelings.
5) There are sex-related differences in attitudes toward math.

Even though there is concensus that sex-related differences in math attitude exist, the magnitude and specific dimensions of these differences are unclear. Suydam and Weaver (1975) quote studies with contradictory results and say that in other studies no significant sex-related differences were found. Aiken states that the correlations between attitude and achievement generally are somewhat higher for girls than

for boys and that significant differences in attitudes are frequently found to favor males over females. Basic agreement with this latter conclusion was reported in the Fennema-Sherman study with learners in grades 6-11 (Fennema-Sherman, Note 1; Sherman-Fennema, Note 2; Fennema and Sherman, Note 3).

The chicken and egg question concerning attitude and achievement is relevant. Do poor attitudes result in poor achievement or vice versa? Undoubtedly there is an interaction between the two and change in one affects the other. However, there is some evidence that variables other than achievement influence attitudes toward mathematics. The influence of parents appears to be significant in the formation of attitudes. The father appears to be the dominant influence. However, a note of caution must be interjected. In 1976 fathers overall have much higher levels of mathematical training and use mathematics more. Since the studies from which the conclusion father influence was drawn did not control mathematical knowledge of the parents, mathematical sophistication is probably the relevant variable instead of the sex of the parent.

Many people believe that teachers' attitudes toward mathematics are important determinants of pupils' attitudes. Some studies support this while others do not. Most studies have been in the elementary school where partitioning teacher effect into one subject area is risky at best. Working with middle school or high school teachers who have selected math as an area of specialty also presents difficulty as long as one is only concerned with the narrow "liking-disliking" definition of attitude. These teachers, because of the similarity of

their mathematical background, undoubtedly show little variance in their liking of mathematics. Overall, the data available suggest that teachers do affect pupil attitude development, both by the teacher's attitude and effectiveness. There also appears to be sex of teacher by sex of pupil interaction.

It must be stressed that all conclusions about attitudes toward math are tentative and based on studies which have not defined "attitude" similarly. In addition, the measuring instruments used undoubtedly did not control for sex-related concerns which may have resulted in biased results. It appears that past research on attitudes toward math is not immediately helpful in understanding or planning interventions to modify sex-related differences in math.

Some insight is gained when a broader definition of feelings toward math is used, i.e., the idea of affective variables as defined by Krathwohl et al. (1964). Affective variables are those which deal with feelings, interests, attitudes, values, and emotional sets or biases. Of this infinite set of variables, those will be discussed which seem helpful in understanding sex-related differences in mathematics study and learning and which may be amenable to change by direct intervention procedures.

Stereotyping Math as a Male Domain

It is commonly accepted that mathematics is stereotyped as an activity more appropriate for males than for females. It has been believed that the sex typing of mathematics as male starts in the elementary school, becomes stronger during adolescent years and is solidified as a male domain by adult years. However, Stein and Smithells (1969) and

Stein (1971) provide evidence that math is not considered masculine by males and females until adolescent years and even during these years is not ranked as highly masculine as are spatial, mechanical and athletic tasks. Bobbe (1971) found that with fourth and sixth grade subjects, arithmetic was judged to be feminine by girls while boys judged it to be appropriate for both sexes. The Fennema-Sherman study indicated that females in grades 6-12 deny that math is a male domain. While the males in the study did not strongly stereotype math as a male domain, at each grade they stereotyped it at significantly higher levels than did females. In the adult world, it is a fact that the use and creation of mathematics is predominantly a male domain. Stein and Smithells (1969) offered evidence that in 12th grade females perceived this fact and were responding to the reality. The females in the Fennema-Sherman study may have been responding to this reality by their overt act of not electing to enroll in mathematics courses.

In conclusion, there is no compelling evidence at the present to conclude that prior to high school mathematics is strongly sex typed. However, during high school years females increasingly behave as if they are aware that mathematics is for males. Throughout high school years, males tend to stereotype math as a male domain at higher levels than do females. Several areas related to the sex typing of math should be investigated.

1) Are certain mathematical areas more sex typed than others? E. g., is problem solving stereotyped as male and computational tasks stereotyped as female?

2) What is the effect on females of males' sex typing mathematics as masculine?

3) Is sex typing of math a stable variable?

Achievement Motivation in Mathematics

Closely related to and affected by stereotyping of math as a male domain is achievement motivation in mathematics. Mathematics is a difficult subject to learn and in order to achieve well in mathematics, one must be highly motivated. Unfortunately, achievement motivation as a construct, has been validated largely by using tasks with arousal conditions which correlate highly with masculine personality characteristics, such as competitiveness and leadership (Stein and Bailey, 1973; Mednick and Weissman, 1975). Because these traits have not been positively associated with the female sex role, females have tended to exhibit them to a lesser degree than have males. As a result, it has been concluded that females have less achievement motivation than do males. However, Stein and her associates (see Stein references) offer an alternative interpretation. They suggest, and offer some data to support the belief, that achievement motivation is exhibited in areas which are stereotyped as appropriate for one's sex. Therefore, according to this theory, if mathematics is perceived as more appropriate for males, then females will have less achievement motivation in this area.

Little is known about the impact on learning mathematics of the psychological construct of achievement motivation. Much is being inferred from psychological studies utilizing subjects' responses made during short time intervals. While such studies result in test-

able hypotheses in a school setting, direct inference about mathematics learning is impossible. Two examples illustrate this point. 1) Stein and her colleagues have written at length on achievement motivation as it relates to sex. The studies rest on a sound theoretical base that achievement motivation is area specific and not a generalized motive. They indicate rather clearly that the sex typing of a subject is related to each sex's achievement motivation in that subject. One method used to assess level of achievement motivation is length of time a subject will pursue a task in an experimental situation. This is labeled persistency on task. Certainly persistency is related to the learning of math. However, the relationship between persistency in a relatively short experimental situation and persistency on a day to day basis over a school year's time is largely unknown--and it is the latter which is important to the learning of mathematics. 2) The major relevant conclusion of the Stein studies, that of sex typing of math by both adolescent boys and girls, was obtained by an experimental task involving an addition problem. There is little addition in high school mathematics. Assuming that learners respond to an addition task in high school in the same way they respond to algebra or geometry tasks is erroneous.

It is unknown how achievement motivation is related to sex-related differences in math. Information concerning several questions would be helpful.

1) Are there sex-related differences in achievement motivation in mathematics? If these differences exist, do they appear at all grade levels?

2) How stable is achievement motivation on a day by day basis in mathematics classes and are there sex-related differences in this stability?

3) Can achievement motivation be improved in a school setting?

Poffenberger and Norton (1963) believe that large societal influences can improve males' achievement motivation. They examined grades from 1955-60, the post Sputnik era, in which there was great hue and cry about Russian superiority in space technology. Male grades showed marked improvement. Poffenberger and Norton attribute this to improved achievement motivation because of the competitiveness arroused in males. If this belief is valid, it might be that one result of the current social climate that is beginning to permit achievement by females will be females' improvement in mathematical skills.

Motive to Avoid Success in Mathematics

Since Horner (1968) first offered evidence that females--particularly high achieving females--fear success in male appropriate areas, much has been written about such a motive as an explanation of females' existing underachievement. In a simplistic way the theory goes something like this: In achievement areas identified as male where females would be competing with males (e.g., medicine, law, business, math) females with the capacity to perform at high levels will not attempt to achieve. They refuse to compete because if they are successful, they fear a variety of sanctions from males. Theoretically the motive to avoid success becomes increasingly salient during high school and college years. Some studies have supported this theoretical construct

while others have not (Mednick and Weissman, 1975). The Fennema-Sherman study did not show that high school females feared success in mathematics any more than males did nor that the motive became stronger as subjects progressed through high school.

It would be profitable to gain more information on types of mathematical activities and the motive to avoid success. Mathematical problem solving and games often involve direct competition. Such activities encourage mathematics learning and provide enjoyment. Do females exhibit a fear of success when directly competing with males? If such a motive inhibits females participation in competitive math activities, their math learning might be inhibited.

Effectance Motivation in Mathematics

One variable, which has been hypothesized to show a sex-related difference, is effectance motivation. This motive can be "inferred specifically from behavior that shows a lasting formalization and that has characteristics of exploration and experimentation" (White, 1959, p. 323). It is closely related to problem solving activity and is often called intrinsic motivation. This motivation would encourage learners to participate in mathematical activities at high cognitive levels. Some believe that females are not so involved in problem solving activities as are males (Carey, 1958; Kagan, 1964). The Fennema-Sherman study found no sex-related difference in this variable at any grade level from 6-12. However, because this type of motivation is so essential to the learning of mathematics further investigation is warranted.

The Confidence-Anxiety Dimension in Mathematics

One tends to do those things one feels confident to do and to avoid activities which arouse anxiety. This confidence-anxiety dimension, as it relates to mathematics learning, is one of the more important affective variables that helps explain sex-related differences in math learning.

The relationship of anxiety and mathematics learning has been explored by a variety of methodologies and with instruments purported to measure debilitative or facilitative anxiety in general and/or specific to mathematics. Callahan and Glennon (1975) conclude that "anxiety and mathematics are related. In general high anxiety is associated with lower achievement in mathematics" (p. 82). Reports from NLSMA indicate that across grade levels 4-10 decreases in facilitating anxiety appeared, with females' scores decreasing more than males' scores. Debilitating anxieties increased for females during these grade levels (Crosswhite, 1972).

Confidence per se has not been given specific attention as it relates to mathematics except in the Fennema-Sherman study. However, self-concept, which appears to be defined in the various scales as self confidence, has received much study. Leviton (1975) and Primavera et al. (1974) reviewed the literature dealing with self-concept and both concluded that a positive relationship exists between academic achievement and self esteem. Brookover and Thomas (1964) offer evidence that self concept is not generalizable but related to specific academic areas. Callahan and Glennon conclude that there is a positive relation-

ship between self-esteem and achievement in mathematics. Others have also recognized the importance of academic self concept in learning mathematics (Bachman, 1970; Fink, 1962).

Although both confidence and anxiety have been defined as separate traits, it appears in relation to mathematics, they are very similar. In the Fennema-Sherman study an attempt was made to measure both confidence and anxiety. A high rating on the confidence scale correlated higher ($r = .89$) with a low rating on the anxiety scale. While it may be possible to talk about the two independently it doesn't appear to be useful.

The literature strongly supports the fact that there are sex-related differences in the confidence-anxiety dimension. It appears reasonable to believe that lesser confidence, or greater anxiety on the part of females is an important variable which helps explain sex-related differences in mathematics studying. Crandall et al. (1962) concluded that girls underestimate their own ability to solve mathematical problems. Others have concluded that females feel inadequate when faced with a variety of intellective, problem solving activities (Kagan, 1964). Maccoby & Jacklin (1973) reported that "girls tend to underestimate their own intellectual abilities more than boys do" (p. 41).

In the Fennema-Sherman study, at each grade level from 6-12, boys were significantly more confident in their abilities to deal with mathematics than were girls. In most instances this happened when there were no significant sex-related differences in mathematics achievement. In addition, confidence in learning mathematics and achievement

were more highly correlated than any other affective variable and achievement (r = .40). Confidence was almost as highly related to achievement as were the cognitive variables of verbal and spatial visualization.

More information is badly needed concerning females' feelings of confidence, what contributes to its development, and possibilities of intervening to change it. At what age should this intervention take place? Currently, there are a number of mathematics anxiety clinics operating at the university level. It appears that, while changes could take place with university age students, it would be more profitable to intervene with much younger students. Mathematics learning is cumulative and if a low level of confidence is operative beginning at sixth grade, the learning that has taken place during the middle and high school years, as well as one's willingness to elect to study math, will have been affected. If females could be prevented from developing feelings of anxiety or lack of confidence, more positive attitudes toward studying math might result and their math learning will be improved.

Usefulness of Mathematics

A different kind of affective variable is belief in the personal usefulness of mathematics. Hilton and Berglünd (1974) and the Fennema-Sherman study provided data indicating that females to a lesser degree than males believe that mathematics is personally useful. However, the difference between male and female beliefs about the usefulness of mathematics was not as great in the Fennema-Sherman study as it was in the Hilton-Berglünd study. This may indicate that the beliefs of females are becoming more similar to males in this aspect.

Other Affective Variables

There are a number of other variables which have been hypothesized to contribute to sex-related differences in mathematics. Several appear to be only symptoms of the basic problem of not electing to study more mathematics. Intervention with these variables would be similar to treating the swelling of jaws when one has the mumps. Such variables include extra curricular activities in mathematics and mathematics related courses studied.

Another variable whose educational implications are unknown is locus of control. Locus of control, as it is relevant here, is the extent to which a person feels responsible for her/his success or failure in academic areas. Ordinarily studies have indicated that females feel they have less control than males do over success or failure and females are more apt to attribute their success or failure to something outside of themselves such as fate or luck (Crandall et al. 1965; Messer, 1972). In addition females expect less success than do males and this appears as early as kindergarten (Frieze, 1975). However, other studies have somewhat countered this trend and have suggested that the perceived sex appropriateness of a task influences whether one feels personal control over the task or expects success. Translating this psychological dimension into implications for education has not yet been done. If investigation is to be done in this area, such translation must receive top priority.

CRITICAL EDUCATIONAL VARIABLES

There are sex-related differences in the final outcome of mathematical education due in large part to females' reluctance--if not refusal--to elect to study mathematics. Some intervention is essential at the present time to ensure equity in mathematical education for both sexes. However, before effective intervention can be planned, more information is needed about critical variables which are amenable to change and important in the educational process.

Learners

The critical question of which females should be the target population of any planned intervention is not easily answered. Females with the intellectual abilities as well as emotional drive to succeed in math related curricula at colleges/universities are one obvious population. It is probably within this population where the greatest success will be realized. At least two other groups of females should also serve as target population: i.e., the precocious and the non-college bound student. There is strong evidence that intervention with the precocious is extremely difficult (Fox, 1975). Perhaps the most difficult group, as well as the group which needs intervention the most, are females from poverty areas and in particular, black females. The NAEP results indicate that the lowest group of achievers on consumer type problems are black females. It is outside the scope of this paper to make decisions concerning which group of females should receive what percentage of attention. In some way this problem must be resolved humanely, rationally and efficiently.

Teachers

Teachers are the most important educational influences on students' learning of mathematics. From kindergarten to high school, learners spend thousands of hours in direct contact with teachers. While other educational agents may have influence on educational decisions, it is the day by day contact with teachers which is the main influence of the formal educational institution. Part of the teachers' influence is in the learners' development of sex role standards. These sex role standards include definitions of acceptable achievement in the various subject areas. It is believed that this influence by teachers is exerted through differential treatment of the sexes as well as expectations of sex-related differences in achievement.

Schonborn (1975) concluded that many studies have indicated teachers treat female and male students differently. In general, males appear to be more salient in the teachers frame of reference. Teachers' interaction with males is greater than it is with females in both blame and praise contacts. Teachers also reinforce in both males and females sexually stereotypic behavior deemed appropriate for their sex (Sears and Feldman, 1966). Brophy and Good have been the major investigators of teacher treatment of males and females. Their well designed studies have been concerned basically with investigation of males' treatment by teachers. In addition to sex of student they often include sex of teacher as a variable. In several studies they have concluded that boys and girls receive equal treatment. However, the data from one of their major studies shows that while the sex of the teacher was

unimportant, high achieving high school boys received significantly more attention in math class than any other group (Good, Sikes & Brophy, 1973). Another study involving first grade reading replicated this trend at nonsignificant levels (Good and Brophy, 1971). One must question why no conclusion was reached about inequitable treatment of high achieving females.

Several unanswered questions remain:

1) What are the effects on achievement and election of math courses of differential teacher treatment and expectations?
2) Do teachers differentially reinforce males and females for specific kinds of mathematical or sexually stereotypic activities. Are males being reinforced more for problem solving activities while females are reinforced for computational activities?
3) What is the effect of sex of teacher on mathematical achievement of boys and girls? While O'Brien (1975) reports no sex of teacher effect Good, Sikes and Brophy (1973), Shinedling and Pedersen (1976) report that male students do best in quantitative scores when taught by male teachers.

School Organization

There is some evidence that schools do influence sexual stereotypes. Minuchin (1971) concluded that children who attended schools categorized as traditional or modern differed in their sex-typed reactions. The interaction of the sexes were different in those schools, also. In the most traditional school boys became leaders in problem solving while girls became followers. This was not so in the less traditional schools. The sex role behavior of children attending traditional schools was more rigid than children attending liberal schools.

Some schools are remarkably more effective in persuading females to attempt high achievement in mathematics. Casserly (1975) identified 13 high schools which had an unusually high percentage of females in advanced placement mathematics and science classes. She concluded that the schools had identified these girls as early as fourth grade and the school teachers and peers were supportive of high achievement by the females. Rowell (1971) pursued the same type of investigation in attempting to identify schools and their characteristics which produced high achieving females in science. Studies identifying and describing those schools which are particularly successful or unsuccessful in encouraging females to enroll in math beyond minimal requirements should be done.

Many are advocating that female only classes will result in equity in mathematical education. The argument for this type of school organization goes something like this. Because peer pressure against female competitiveness is too strong a force, females will not compete against

males in a mixed sex classroom. Female leadership (in problem solving in this case) is only able to emerge when competition with males is eliminated. Teachers will not have different sex-related expectations and behaviors if only one sex is present. Single sex classrooms appear to provide a simple solution to a complex problem. However, the weight of evidence found does not support this type of grouping. Conway (1973) convincingly argues that throughout history separate education for the sexes has resulted in inferior education for females. Keeves, (1943) after a careful and thorough review of mathematics and science education in 10 countries, concluded that the "extent to which a community provides for education in single sex schools would appear to indicate the extent to which it sees its boys and girls requiring different preparation for different societal roles" (p. 52). He argues that "in so far as a community has different expectations for different groups of its members and proceeds to mould its future members through different organizations, then it fails to provide equal opportunities for individual development" (p. 52). In an unreported study comparing attitudes of 10th grade females who had spent most of their educational life in single or mixed sex classrooms, females from the mixed sex classrooms exhibited significantly more positive attitudes toward mathematics (Fennema-Meyer, Note 4).

Before single sex classrooms are embraced as a panacea for females' educational equity, careful examination must be done concerning long term effectiveness of such programs. In reality, this may be a partially non researchable problem. No one can foresee the implications for

females 50 years from the present time of being isolated in their mathematical training. Because of what has happened to females as well as blacks over the last century, single sex classroom school organization must be approached with caution.

CONCLUSIONS

Although many believe that males achieve at higher mathematical levels that do females, this is not true for all groups of females and males. Few sex-related differences in achievement are evident before high school. When the number of years of mathematics study is equated, few differences in achievement are found during or after high school. It is true that more females than males elect not to enroll in math courses during high school or post high school years. The critical problem in providing equity of mathematical education for the sexes is to make sure that equal numbers of females and males study mathematics beyond minimum requirements.

Although this is not a simple task, the first step has been taken, that is, national recognition of the problem. Furthermore, there is enough convincing evidence from recent research to suggest that educational policies in the schools should be able to encourage more female participation in mathematics. We also have enough leads from recent research to suggest that further research efforts could be extremely fruitful in helping us to better understand differences in participation and in suggesting alternative policies for making participation in mathematics more equitable.

It is important to realize we are beginning to understand the cognitive, affective and educational variables that are associated with sex-related differences in course election and mathematics learning. For example, with the possible exception of spatial visualization, cognitive variables do not seem helpful in understanding sex-related differences in math. More information about the interrelationships of spatial visualization skills and mathematics learning is essential because of the increasing reliance on spatial attributes in school math programs and the strong evidence that there are sex-related differences in spatial visualization. Particularly critical is the question of how spatial skills are used in mathematical problem solving and if this use differs by sex.

Research also implies that affective variables appear crucial in explaining sex-related differences in mathematics study. Although much of the previous research which has investigated attitudes toward math is flawed, present literature indicates that two affective variables are particularly important: stereotyping math as a male domain and confidence or anxiety in learning math. In addition, sex-related differences are apparent in other affective variables such as achievement motivation in mathematics and perceived usefulness of mathematics. More information is critically needed on developmental patterns of affective variables, stability of such variables and effective ways of increasing positive attitudes toward mathematics.

Educational variables which appear helpful in understanding and ultimately eliminating sex-related differences in mathematics can be

categorized into variables dealing with learners, teachers and school organization. Although little information is available that is precisely concerned with the relationship between these variables and sex-related differences in math, several tentative conclusions may be stated: 1) Subgroups of females may differ in cognitive and affective variables related to mathematics and may require different educational treatments; 2) Since teachers treat females and males differently, it may follow that this differential treatment affects the willingness of females and males to study math; 3) Because of possible long term ramifications, a mathematics program organized to provide single sex instruction must be approached with extreme caution.

In conclusion, there is much we now know from research about reasons for the unequal participation of females and males in mathematics. First, we know there are differences in mathematics study and the time this inequity becomes apparent has been identified (e.g., differentiation begins to be observed as more educational and personal choices are available). Second, we know that there is little difference between males and females in most cognitive variables except for visual spatial skills. Third, we not only know that attitudes are important to the choices being made by individual females and that these attitudes come from a variety of sources such as the society, family, teachers, schooling and peers but some attitudes have been identified which are related to ongoing participation in mathematics and which discriminate between males and females. Fourth, we also know that there exist some programs that have been successful in changing attitudes. This information also

suggests what we still need to know to be more effective in changing females participation in mathematics. For example, we need more information about when and what affects crucial female attitudes towards mathematics; we must identify those sources that may be most susceptible to change; we need to know more about the comparative effectiveness of certain measures to change attitudes of different females towards mathematics; we need to better understand the real impact of the observed cognitive differences between males and females and the relationships between verbal skills, visual skills and mathematical learning; we need to know more about the actual procedures being used in school mathematics instruction that directly result in inequitable participation in mathematics by the sexes; and we need more examples of successes and failures in actually inducing more female participation in mathematics.

RECOMMENDATIONS

The body and conclusions of this paper suggest certain recommendations for funding focused on improving females' participation in mathematics. These recommendations are directed toward three areas: 1) Directions for research, 2) Intervention or demonstration projects and 3) General considerations to optimize the chances that the money spent will accomplish its goals, i.e., to improve females' participation in mathematics as soon as possible and to ensure that females will continue to be involved with mathematics.

Directions for Research

1. Well designed studies which will provide knowledge about variables related to females' decisions to study or not study mathematics beyond minimum requirements should receive highest priority. Each study funded should deal with a population of specified characteristics so important conclusions can be generalized to groups with similar characteristics. Since a tremendously complex interplay of affective, educational, societal and possibly cognitive variables affect the decision to study mathematics, many different types of studies are necessary.

2. Studies funded should deal directly with mathematics instruction and learning. The importance of investigations made within the realm where behavior is to be changed can not be over emphasized. Results of such investigations are immediately generalizable and will provide needed information for planning effective intervention programs to increase females' participation in mathematics. The projects to be funded should rest on sound theoretical foundations with translation into action or deeper understanding of the educational realm as the goal.

3. Some studies funded should deal directly with the relation between verbal, visual and mathematical skills, especially as they relate to male-female differences. Stability of relations across many developmental levels as well as patterns of development among these cognitive variables should be investigated.

Intervention Projects

1. Carefully described action programs designed to increase specific target populations of women's participation in mathematics should be funded. Such programs must have accurately described features and well defined target populations so that the programs can be replicated with similar populations. There are currently many intervention programs in operation and brief descriptions of some of these appear in Appendix B. However, there is practically no generalizable knowledge about effective intervention, except for the fact that it is extremely difficult. Each program funded should rest on sound theoretical bases, be designed for well identified sub-populations and have well described features. Any intervention must be planned considering many affective and cognitive variables in the complex environment of schools. Learner characteristics, teacher characteristics, mathematics content, instructional method and classroom organization must be considered.

2. Each intervention project which is funded should include plans for extensive evaluation. Included must be long term evaluation as well as short term evaluation. A permanent change in behavior of females toward mathematics must be the goal of any intervention program and this change can not be assessed immediately following participation in such a program. Critical long range evaluation points should be identified. Plans for gathering data at those points must be included and funds must be made available to ensure that the true effect of interventions can be ascertained. In addition plans for evaluation should include provision for assessing unintended as well as intended consequences.

3. Projects funded should originally be small scale programs. These programs should be located in a realistic field setting so that their effectiveness and social impact and transferability to larger projects can be ascertained.

4. Projects funded should include a diversity of target groups in addition to the females directly involved. The opinions of parents, teachers, counselors and in particular male peers make a direct contribution to a female's decision to study mathematics. Unless such significant others' opinions of females as learners of mathematics is positive, it will be extremely difficult, if not impossible, to change females' traditional patterns of behavior toward mathematics.

Basic Considerations

1. Interdisciplinary teams of scholars as well as practitioners should be actively involved in funding decisions as well as funded projects. Two groups of scholars must be specifically involved, i.e., those concerned with the study of women and mathematics educators. Mathematics education is a specialized field which includes research scientists as well as developers and implementers of curriculum. People from this field can most effectively investigate current mathematics learning as well as make effective plans for changing mathematics curricula. However, mathematics education scholars must be joined by scholars who have demonstrated their concern for achieving equity for women. It is this latter group who will be particularly helpful in identifying specific areas of concern.

In addition, scholars from other disciplines such as psychology and sociology would provide breadth of knowledge which would increase the possibility of change. Since the major changes which will achieve equity in mathematics for females will have to take place in schools, practitioners who understand the parameters of effecting educational change must also be involved in decision making and carrying out of all projects.

2. A substantial portion of funding should be available for longitudinal studies and long term projects. The importance of longitudinal studies for understanding sex-related differences in mathematics and their related variables is evident. Such studies could provide information about stability of important variables within an individual as well as identify which variables are of prime saliency at various developmental points. Long term evaluation of all projects should be included.

3. Chains of related inquiry must be funded. All to often, both in mathematics education and research on women, isolated studies and projects are done. Isolated pieces of information that result from isolated activities will not effectively help women gain equity in mathematics. Only when profitable studies and effective projects are followed by additional studies and projects will we build enough knowledge to effect change.

4. Studies and projects using nontraditional as well as traditional methodologies should be funded. Some studies funded should utilize case study, clinical or participant observation methodologies. Such methodologies emphasize the opportunities for generating new hypotheses and

knowledge. The subtle and difficult to measure hidden curriculum, that says mathematics is not for females, has not and can not be identified with traditional research methodologies. Observational and clinical strategies are needed in order to locate and describe communication patterns and messages from peer to peer, teacher to learner, and curriculum to learner. Because we know that behavior is extremely difficult to change, innovative ways to change female's behavior in mathematics must be encouraged by funding.

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